

The majority of the content on this test is non-calculator, with the exception of #14-17.

Factor each polynomial completely. You **do not** need to solve for x.

1) $64x^3 - 27$ 2) $10x^3 + 12x^2 + 2x$ 3) $4x^3 - 100x^2$
 $(4x-3)(16x^2+12x+9)$ $2x(5x^2+6x+1)$ $4x^2(x-25)$
 [Diff of Cubes] $2x(5x+1)(x+1)$
 $5 \begin{array}{r} 5 \\ \times 6 \\ \hline 1 \end{array}$

Finding Solutions by Factoring

For each equation below, find all real solutions by factoring and solving for x:

4) $5x^3 - 20x = 0$ 5) $3x^3 + 15x^2 = 72x$ 6) $35x^2 + 15x - 20 = 0$
 $5x(x^2 - 4) = 0$ $3x^3 + 15x^2 - 72x = 0$ $5(7x^2 + 3x - 4) = 0$
 $5x(x+2)(x-2) = 0$ $3x(x+8)(x-3) = 0$ $5(7x-4)(x+1) = 0$
 $x = 0, -2, 2$ $x = 0, -8, 3$ $x = \frac{4}{7}$ or $x = -1$

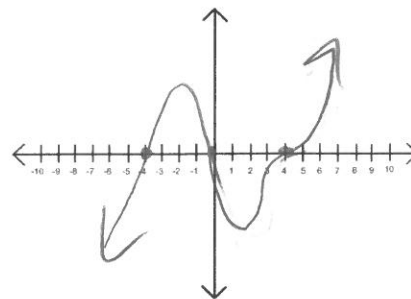
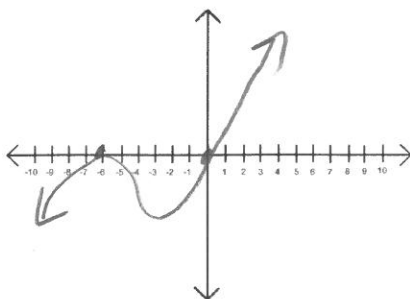
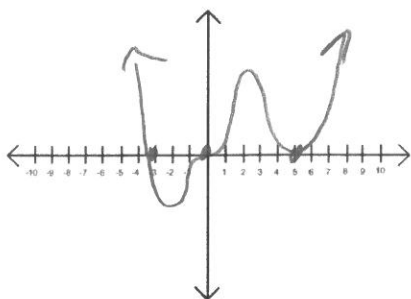
Convert each function from factored form to standard form. Rewrite each polynomial function in standard form.

7) $y = (x-1)(x+2)^2$ 8) $y = (x-1)^2(x-3)$ 9) $y = -x(x+3)^2$
 $y = (x-1)(x^2+4x+4)$ $y = (x^2-2x+1)(x-3)$ $y = -x(x^2+6x+9)$
 $y = x^3 + 4x^2 + 4x - x^2 - 4x - 4$ $y = x^3 - 2x^2 + x - 3x^2 + 6x + x - 3$ $y = -x^3 - 6x^2 - 9x$
 $y = x^3 + 3x^2 - 4$ $y = x^3 - 5x^2 + 7x - 3$

Sketching Polynomials

Sketch the graph of each polynomial to clearly show the end behavior and behavior at each root. **Do NOT** worry about accurately showing the location of the local maximums or minimums.

10) $f(x) = x^3(x+3)(x-5)^2$ 11) $f(x) = 4x(x+6)^2$ 12) $f(x) = 2x(x+4)(x-4)^3$
 x-intercepts: $(0,0)(-3,0)(5,0)$ x-intercepts: $(0,0)(-6,0)$ x-intercepts: $(0,0)(-4,0)(4,0)$
 Degree: 6 Degree: 3 Degree: 5
 LC: pos (1) LC: pos (4) LC: pos (2)
 End Behavior: ↖ ↗ End Behavior: ↘ ↗ End Behavior: ↘ ↗

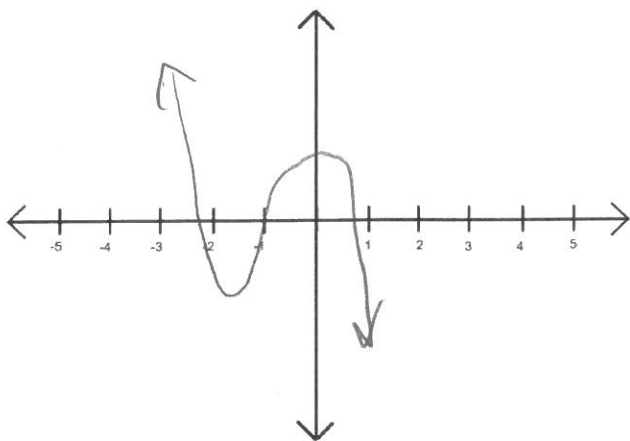


Knowledge of Polynomial Graphs

13) Provide the information requested for each polynomial function:

- a) $y = -11x^7 - 6x^4 - 7x^2 + 7$ the leading coefficient – positive / negative -11
- b) $y = 8x^9 - 3x^{12} - 5x^6 + 2$ end behavior ↘ ↘
- c) $y = -5x^2(x+9)(x-7)^2$ the x-intercepts (0,0) (-9,0) (7,0)
- d) $y = 4x(x-12)^5(x-4)^2$ multiplicity for the factor 4 2
- e) $y = 3x^2(x-5)^2(x-3)^3$ at 3, the graph will cross / bounce / wiggle wiggle
- f) $y = -4x^6 + 3x^2 - 8x + 12$ the degree – number and odd / even 6 (even)
- g) $y = 9x^2 - 12x - 22$ the y-intercept (0, -22)
- h) $y = 9x + 8x^2 - 13x^7 - 10$ the leading coefficient – positive / negative -13
- i) $y = x^3(x+4)(x-2)^2(x-7)$ the degree – number and odd / even 7 (odd)
- j) $y = (x-4)^4(x+5)^3(x-3)^2$ end behavior ↘ ↗

14) **Graph the Polynomial:** $f(x) = -x^5 - 3x^4 - 2x^3 + x + 1$ on a calculator and find the following features.



- Degree: 5
- # of Solutions: 3
- Max # of Turning Pts: 2
- Positive or Negative Leading Coefficient? (circle one)
- End Behavior: ↗ ↘
- x-intercepts: (-2.11, 0) (-1, 0) (0.72, 0)
- y-intercept: (0, 1)
- Local Maximums: (.31, 1.22)
- Local Minimums: (-1.75, -1.75), (mistake only)
- Domain: (-∞, ∞)
- Range: (-∞, ∞)
- Increasing: (-1.75, .31)
- Decreasing: (-∞, -1.75) ∪ (0.31, ∞)

15) Refer to the data table:

- a. Show how you can use finite differences to determine the degree of the polynomial that fits the data.

4th degree
Quartic

x	0	1	2	3	4	5
y	2	0	6	56	210	552

1st: -2, 6, 50, 154, 342
 2nd: 8, 44, 104, 188
 3rd: 36, 60, 84
 4th: 24, 24

- b. Use the regression feature of your calculator to write the polynomial function for this relationship.

$$y = x^4 - 3x^2 + 2$$

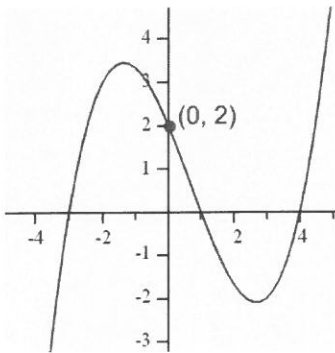
Writing Equations

- 16) Write a polynomial function $f(x)$ of **least degree** that has a leading coefficient of 1, and the given zeros: -2, 5, 3

$$y = (x + 2)(x - 5)(x - 3)$$

- 17) Write an equation of the following polynomial. Multiply factors to write the equation in **standard form** (don't forget about a).

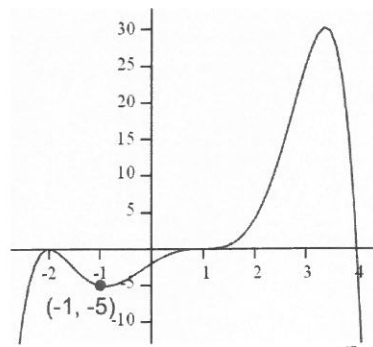
$$y = \frac{1}{6}x^3 - \frac{1}{3}x^2 - \frac{11}{6}x + 2$$



$$\begin{aligned} y &= a(x + 3)(x - 1)(x - 4) \\ 2 &= a(0 + 3)(0 - 1)(0 - 4) \\ 2 &= a(3)(-1)(-4) \\ 2 &= 12a \quad (\div \text{ both sides by } 12) \\ a &= \frac{1}{6} \\ y &= \frac{1}{6}(x + 3)(x - 1)(x - 4) \\ y &= \frac{1}{6}(x^2 + 2x - 3)(x - 4) \\ y &= \frac{1}{6}(x^3 - 4x^2 + 2x^2 - 8x - 3x + 12) \\ y &= \frac{1}{6}(x^3 - 2x^2 - 11x + 12) \\ y &= \frac{1}{6}x^3 - \frac{1}{3}x^2 - \frac{11}{6}x + 2 \end{aligned}$$

- 18) Write an equation of the following polynomial. You may leave it in **factored form** (don't forget about a).

$$y = -\frac{1}{8}(x + 2)^2(x - 1)^3(x - 4)$$



$$\begin{aligned} y &= a(x + 2)^2(x - 1)^3(x - 4) \\ -5 &= a(-1 + 2)^2(-1 - 1)^3(-1 - 4) \\ -5 &= a(1)^2(-2)^3(-5) \\ -5 &= a(1)(-8)(-5) \\ -5 &= 40a \\ \frac{-5}{40} &= \frac{40a}{40} \\ a &= -\frac{1}{8} \end{aligned}$$