Truth Tables and Logical Equivalence HW:

1. Use truth tables to establish the following logical equivalences:

a $\neg(\neg p) = p$ **b** $p \wedge p = p$ **c** $p \vee (\neg p \wedge q) = p \vee q$

d $\neg(p \veebar q) = p \veebar \neg q$ **e** $\neg(q \vee \neg p) = \neg q \wedge (p \vee q)$ **f** $\neg p \veebar (p \vee q) = p \vee \neg q$

2.a Complete the truth table below:

p	q	$p \veebar q$	$q \wedge (p \veebar q)$	$(p \veebar q) \vee p$
T	T			
T	F			
F	T			
F	F			

b Consider the propositions p: $-3 \leqslant x \leqslant 7$ and q: $x \geqslant 2$.
Find the values of x which make the following propositions true:

i $p \veebar q$ **ii** $q \wedge (p \veebar q)$ **iii** $(p \veebar q) \vee p$

3. Construct truth tables for these compound statements:

a $\neg p \vee (q \wedge r)$ **b** $(p \vee \neg q) \wedge r$ **c** $(p \vee q) \vee (p \wedge \neg r)$

4. Determine whether the following propositions are tautologies, logical contradictions, or neither:

a $(p \vee q) \vee \neg(r \wedge p)$ **b** $(p \veebar r) \wedge \neg q$ **c** $(q \wedge r) \wedge \neg(p \vee q)$

5. **a** Consider the propositions

p: Jake owns a phone
q: Jake owns a TV
r: Jake owns a laptop.

Write down the meaning of:

i $p \wedge q$ **ii** $(p \wedge q) \wedge r$ **iii** $q \wedge r$ **iv** $p \wedge (q \wedge r)$

b Use truth tables to show that $(p \wedge q) \wedge r = p \wedge (q \wedge r)$.

6. Use truth tables to show that $(p \vee q) \vee r$ and $p \vee (q \vee r)$ are logically equivalent.

7. **a** Consider the propositions

p: Mary will study Mathematics next year
q: Mary will study French next year
r: Mary will study German next year.

Write down the meaning of:

i $q \vee r$ **ii** $p \wedge (q \vee r)$ **iii** $p \wedge q$ **iv** $p \wedge r$ **v** $(p \wedge q) \vee (p \wedge r)$

b Use truth tables to show that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

8. **a** Use truth tables to show that $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$.

b Consider the Venn diagram alongside, where P, Q, and R are the truth sets of p, q, and r respectively.

On separate Venn diagrams, shade the truth set for:

i $p \vee (q \wedge r)$

ii $(p \vee q) \wedge (p \vee r)$

Comment on your results.