Optimising HW:

1. A small business which employs x workers earns a profit given by
$P(x) = -x^3 + 300x + 1000$ pounds.
How many workers should be employed to maximise the profit?

2. The total cost of producing x blankets per day is $(\frac{1}{4}x^2 + 8x + 20)$ dollars, and for this production level each blanket may be sold for $(23 - \frac{1}{2}x)$ dollars.
How many blankets should be produced per day to maximise the total profit?

3. For the cost function $C(x) = 720 + 4x + 0.02x^2$ dollars and revenue function $R(x) = 15x - 0.002x^2$ dollars, find the production level that will maximise profits.

4. An open rectangular box has a square base, and a fixed inner surface area of 108 cm^2.

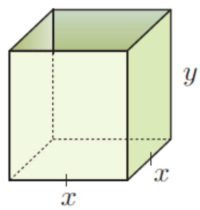
 a Explain why $x^2 + 4xy = 108$.

 b Hence show that $y = \dfrac{108 - x^2}{4x}$.

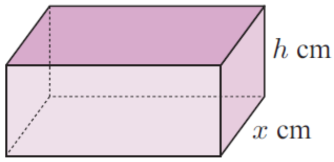
 c Find a formula for the capacity C of the container, in terms of x only.

 d Find $\dfrac{dC}{dx}$. Hence find x when $\dfrac{dC}{dx} = 0$.

 e What size must the base be in order to maximise the capacity of the box?



5. Radioactive waste is to be disposed of in fully enclosed lead boxes of inner volume 200 cm^3. The base of the box has dimensions in the ratio $2:1$.



a What is the inner length of the box?

b Explain why $x^2h = 100$.

c Explain why the inner surface area of the box is given by $A(x) = 4x^2 + \dfrac{600}{x}$ cm^2.

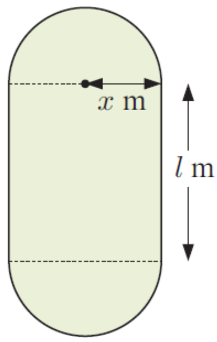
d Use technology to help sketch the graph of $y = 4x^2 + \dfrac{600}{x}$.

e Find $\dfrac{dA}{dx}$. Hence find x when $\dfrac{dA}{dx} = 0$.

f Find the minimum inner surface area of the box.

g Sketch the optimum box shape, showing all dimensions.

6. An athletics track has two 'straights' of length l m and two semi-circular ends of radius x m. The perimeter of the track is 400 m.

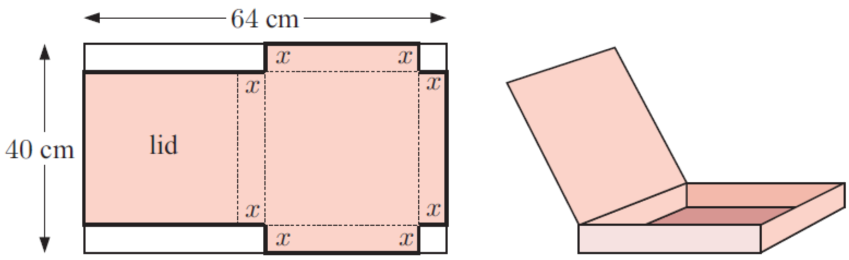


a Show that $l = 200 - \pi x$ and hence write down the possible values that x may have.

b Show that the area inside the track is given by $A = 400x - \pi x^2$ m^2.

c What values of l and x produce the largest area inside the track?

7. A closed pizza box is folded from a sheet of cardboard 64 cm by 40 cm. To do this, equal squares of side length x cm are cut from two corners of the short side, and two equal rectangles of width x cm are cut from the long side as shown.



a Find the dimensions of the lid and the base of the box in terms of x.

b Find the volume of the box in terms of x.

c What is the maximum possible volume of the box?

d What are the dimensions of the box which has the maximum volume?