Date: 3/15/2017

Title: Optimisation

Beware the Ides of March!

Obj: To find the maximum or minimum value of a function.

1)Let's Go Over hw

2)Do #7-10

3) Notes on Optimization

4)Do/start hw

5) Review next class and test is the 21st

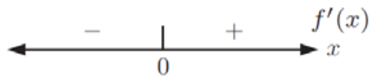
Homework Check:

1. **a** 4 **b** **i** $(-2, 9)$ **ii** $(2, -6)$

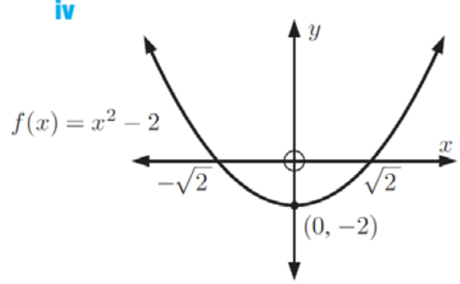
c **i** 11, when $x = 6$ **ii** -10, when $x = 4$

d 9, when $x = -2$ **e** -6, when $x = 2$

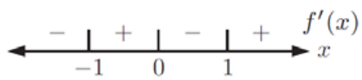
2. **a** **i** $f'(x) = 2x$ **ii**



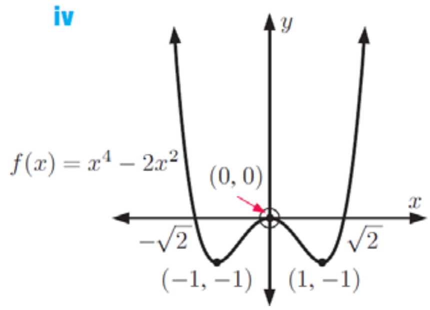
iii local minimum at $(0, -2)$ **iv**



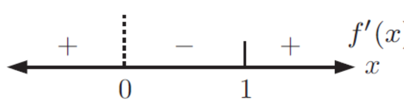
d **i** $f'(x) = 4x^3 - 4x$ **ii**



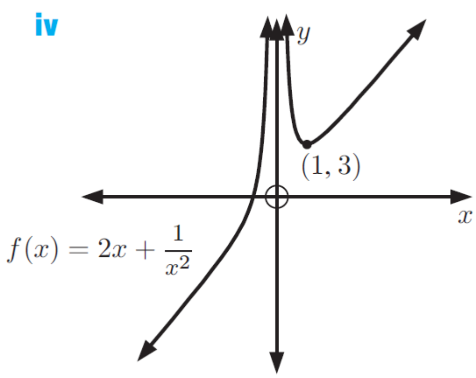
iii local maximum at $(0, 0)$, local minima at $(-1, -1)$ and $(1, -1)$ **iv**



g **i** $f'(x) = 2 - \dfrac{2}{x^3}$ **ii**



iii local minimum at $(1, 3)$ **iv**



3. $a = 9$

4. **a** $a = -12,\ b = -13$ **b** $f'(x) = 3x^2 - 12$
c local maximum at $(-2, 3)$, local minimum at $(2, -29)$

5. **a** $C'(x) = 0.0021x^2 - 0.3592x + 14.663$
b $x \approx 67.3,\ x \approx 104$
c Minimum hourly cost: \$529.80 when 104 hinges are made.
Maximum hourly cost: \$680.95 when 150 hinges are made.

6. **a** $\frac{dM}{dt} = 3t^2 - 6t$ **b** $\frac{dR}{dt} = 8t + 4$

7. **a** $B'(t) = 0.6t + 30$ thousand per day
$B'(t)$ is the instantaneous rate of growth of the bacteria.
b $B'(3) = 31.8$
After 3 days, the bacteria are increasing at a rate of 31.8 thousand per day.
c $B'(t)$ is always positive for $0 \leqslant t \leqslant 10$,
$\therefore$ $B(t)$ is increasing for $0 \leqslant t \leqslant 10$.

8. **a** 1.2 m
b $s'(t) = 28.1 - 9.8t$ This is the speed of the ball (in m s^{-1}).
c $t \approx 2.87$ s. The ball has reached its maximum height.
d 41.5 m
e **i** $28.1\ \text{m s}^{-1}$ **ii** $8.5\ \text{m s}^{-1}$ **iii** $-20.9\ \text{m s}^{-1}$
The sign tells us whether the ball is travelling upwards $(+)$ or downwards $(-)$.
f 5.78 s

9. **a** Near part is 2 km from the shore line, far part is 3 km away.
b $\frac{dy}{dx} = \frac{3}{10}x^2 - x + \frac{3}{5}$
At $x = \frac{1}{2}$, $\frac{dy}{dx} = 0.175$. The gradient of the hill at a point 500 m from the shoreline is 0.175 (going uphill).
At $x = 1\frac{1}{2}$, $\frac{dy}{dx} = -0.225$. The gradient of the hill at a point 1.5 km from the shoreline is 0.225 (going down).
c 2.55 km from the sea. The depth is 0.0631 km (63.1 m).

10. **a** $R(x) = 70x$
b $P(x) = -0.0002x^3 - 0.04x^2 + 60x - 3000$
c $P'(x) = -0.0006x^2 - 0.08x + 60$
d $P'(120) = 41.76$ This means that producing 121 items instead of 120 increases profit by about \$41.76.

Optimisation:

There are many problems for which we need to find the **maximum** or **minimum** value for a function. We can often solve such problems using differential calculus techniques. The solution is referred to as the **optimum solution**, and the process is called **optimisation**.

The maximum or minimum value does not always occur when the first derivative is zero. It is essential to also examine the values of the function at the endpoints of the domain for global max and min.

Steps to Optimising:

1. If necessary, draw a large, clear diagram of the situation.

2. Construct a formula with the variable to be optimised as the subject. It should be written in terms of a single variable such as x. You should write down what restrictions there are on x.

3. Find the **first derivative** and find the values of x where it is **zero**.

4. If there is a restricted domain such as $a \le x \le b$, the maximum or minimum may occur either when the derivative is zero or else at the endpoint.

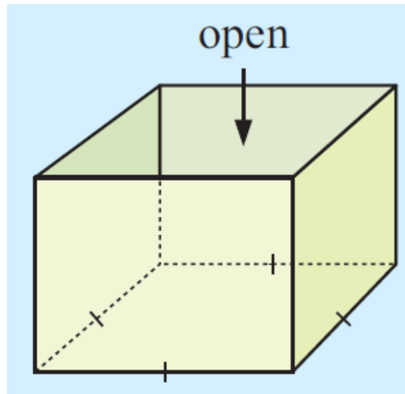
Show using the **sign diagram test** or the **graphical test**, that you have a maximum or a minimum.

Example 1: The cost of making x tennis racquets each day is given by $C(x) = x^2 - 20x + 120$ dollars per racquet.

How many racquets should be made per day to minimise the cost per racquet?

Example 2: When a stone is thrown vertically upwards, its height above the ground is given by $h(t) = 49t - 9.8t^2$ meters. Find the maximum height reached.

3. A 4 litre container must have a square base, vertical sides, and an open top. Find the most economical shape which minimizes the surface area of material needed.

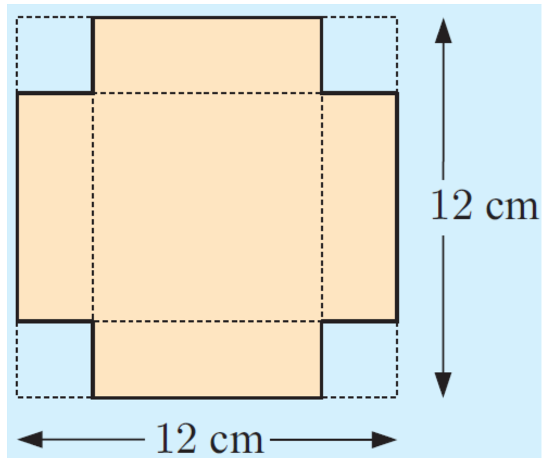


What are we looking for? minimising surface area

What do we have? volume - 4 litre container

4. A square sheet of metal 12 cm x 12 cm has smaller square cut from its corners as shown.

What sized square should be cut out so that when the sheet is bent into an open box it will hold the maximum amount of liquid?



Optimising HW:

1. A small business which employs x workers earns a profit given by $P(x) = -x^3 + 300x + 1000$ pounds.
How many workers should be employed to maximise the profit?

2. The total cost of producing x blankets per day is $(\frac{1}{4}x^2 + 8x + 20)$ dollars, and for this production level each blanket may be sold for $(23 - \frac{1}{2}x)$ dollars.
How many blankets should be produced per day to maximise the total profit?

3. For the cost function $C(x) = 720 + 4x + 0.02x^2$ dollars and revenue function $R(x) = 15x - 0.002x^2$ dollars, find the production level that will maximise profits.

4. An open rectangular box has a square base, and a fixed inner surface area of 108 cm^2.

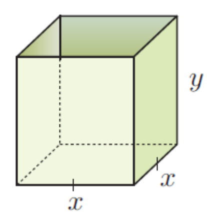
   a Explain why $x^2 + 4xy = 108$.

   b Hence show that $y = \dfrac{108 - x^2}{4x}$.

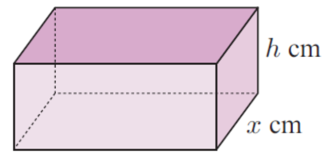
   c Find a formula for the capacity C of the container, in terms of x only.

   d Find $\dfrac{dC}{dx}$. Hence find x when $\dfrac{dC}{dx} = 0$.

   e What size must the base be in order to maximise the capacity of the box?



5. Radioactive waste is to be disposed of in fully enclosed lead boxes of inner volume 200 cm^3. The base of the box has dimensions in the ratio 2 : 1.



a What is the inner length of the box?

b Explain why $x^2h = 100$.

c Explain why the inner surface area of the box is given by $A(x) = 4x^2 + \frac{600}{x}$ cm^2.

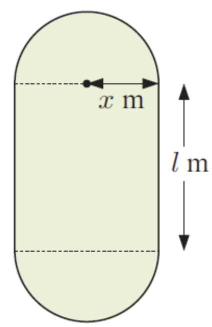
d Use technology to help sketch the graph of $y = 4x^2 + \frac{600}{x}$.

e Find $\frac{dA}{dx}$. Hence find x when $\frac{dA}{dx} = 0$.

f Find the minimum inner surface area of the box.

g Sketch the optimum box shape, showing all dimensions.

6. An athletics track has two 'straights' of length l m and two semi-circular ends of radius x m. The perimeter of the track is 400 m.

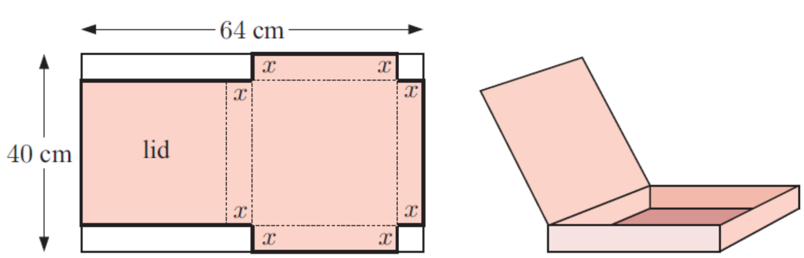


a Show that $l = 200 - \pi x$ and hence write down the possible values that x may have.

b Show that the area inside the track is given by $A = 400x - \pi x^2$ m^2.

c What values of l and x produce the largest area inside the track?

7. A closed pizza box is folded from a sheet of cardboard 64 cm by 40 cm. To do this, equal squares of side length x cm are cut from two corners of the short side, and two equal rectangles of width x cm are cut from the long side as shown.



a Find the dimensions of the lid and the base of the box in terms of x.

b Find the volume of the box in terms of x.

c What is the maximum possible volume of the box?

d What are the dimensions of the box which has the maximum volume?