# 1) check your answers on the homework <br> 2) new notes on natural logarithms 

3) lots of practice because it makes perfect

- and yes - finish for homework..


## check homework

Name: $\qquad$ Unit 7: Exponential \& Logarithmic Functions $\square$
Date: $\qquad$ Bell: $\qquad$ Homework 6: Solving Logarithmic Equations
** This is a 2-page document **


| $\text { 9. } \begin{gathered} \log _{4}\left(2 m^{3}-14 m^{2}\right)-\log _{4}(2 m)=\log _{4} 8 \\ \log _{4}\left(\frac{2 m^{3}-14 m^{2}}{2 m}\right)=\log _{4} 8 \\ m^{2}-7 m=8 \\ m^{2}-7 m-8=0 \\ (m-8)(m+1)=0 \\ m=8 \end{gathered} m=m$ | 10. $2 \cdot \log (x-3)=\log 25$ $\begin{gathered} \log (x-3)^{2}=\log 25 \\ x^{2}-6 x+9=25 \\ x^{2}-6 x-16=0 \\ (x-8)(x+2)=0 \\ x x=8 \end{gathered}$ |
| :---: | :---: |
| $\text { 11. } \begin{aligned} & \log _{3}(2 x-7)=4 \\ & 3^{4}=2 x-7 \\ & 81=2 x-7 \\ & 88=2 x \\ & 44=x \end{aligned}$ | 12. 10 $\begin{gathered} \log _{8}(28 k-20)+15=18 \\ \log _{8}(28 k-20)=3 \\ 8^{3}=28 k-20 \\ 512=28 k-20 \\ 532=28 k \\ 19=k v \end{gathered}$ |
| $\text { 12. } \begin{aligned} & \log _{9}(15-4 n)=\frac{1}{2} \\ & 9^{1 / 2}=15-4 n \\ & 3=15-4 n \\ &-12=-4 n \\ & 3=n \end{aligned}$ | $\text { 14. } \begin{gathered} \log _{2} 4+\log _{2}(c-9)=5 \\ \log _{2}(4(c-9))=5 \\ 2^{5}=4 c-36 \\ 32=4 c-36 \\ 68=4 c \\ 17=c \end{gathered}$ |
| $\begin{aligned} & 15 \cdot 2 \cdot \log _{4} k=4 \\ & \log _{4} k^{2}=4 \\ & 4^{4}=k^{2} \\ & 256=k^{2} \\ & 0=k^{2}-256 \\ & 0=(k+16)(k-16) \end{aligned}$ | 16. $\begin{aligned} \log _{8}\left(p^{2}+15\right) & =2 \\ 8^{2} & =p^{2}+15 \\ 64 & =p^{2}+15 \\ 49 & =p^{2} \\ 0 & =p^{2}-49 \\ 0 & =(p+7)(p-7) \\ & p=-7)(p=7 \end{aligned}$ |

# Let's Learn about The Natural Logarithm! 

It's easy and more something to memorize for now until we get to applications next week!

## In your notebook:

## Base $e$ and Natural Logs (ln)

$$
\begin{array}{ll}
\text { Natural Base }(e): \quad e \approx 2.71828 \ldots . & \begin{array}{l}
\text { Find } e \text { on your calculator.. } \\
(2 \text { places })
\end{array} \\
\text { Natural Base Exponential Function: } & \left.y=e^{x} \quad \text { (inverse of } \ln \right)
\end{array}
$$

$$
\text { Natural Logarithm (ln): } \left.\quad \log _{e} x=\ln x \quad \text { (inverse of } e\right)
$$

Example 1 Evaluate Natural Bases
a) $e^{2}$
(ㅇㅇㅇㅇㅇ:

Julie:Erase the smileys to reveal answer
Example 2 Evaluate Natural Logs
a) $\ln (4)$
b) $\ln (0.05)$

b) $e^{-1.5}$

이응ㅇ

| Use your <br> calculator to <br> see if you get <br> the same <br> answer.${ }^{2}$ |
| :--- |

# Notice the In key on your calculator (that is the log base e) 

Today's packet is much of what we've been doing (all the same rules and techniques apply - but with base e (natural logarithms :)
pass out and go over the next 2 pages as much as they need from you.

| Name: |  |  | Date: |  |
| :---: | :---: | :---: | :---: | :---: |
| Topic: |  |  | Class: |  |
| Main Ideas/Questions | Notes/Examples |  |  |  |
| What is "e"? | - $e$ is an $\qquad$ with an approximate value of $\qquad$ <br> - $e$ often occurs as the base of exponential and logarithmic functions that describe real-world scenarios. |  |  |  |
| Base "e" Exponential Functions | - Exponenti exponenti <br> - Example: | with base $e$ | re ca |  |
| Base "e" Logarithmic Functions | - Logarithmic functions with base $e$ are called $\qquad$ <br> - Example: $\qquad$ . This is often abbreivated as $\qquad$ |  |  |  |
| Converting Between Forms | Write each equation in logarithmic form. |  |  |  |
|  | 1. $e^{x}=24$ | 2. $e^{9}=x$ |  | 3. $e^{x+5}=72$ |
|  | Write each equation in exponential form. |  |  |  |
|  | 4. $\ln x=58$ | 5. $\ln 6=x$ |  | 6. $\ln (x-9)=32$ |
| Simplifying with Properties | Condense each expression into a single logarithm. |  |  |  |
|  | 7. $\ln 3+\ln 16$ | 8. In 63-2 | In 3 | 9. $\frac{1}{3} \cdot \ln 64+2 \cdot \ln x$ |
|  | Expand each logarithm. |  |  |  |
|  | 10. $\ln 5 x$ | 11. $\ln \left(\frac{a^{3}}{b}\right)^{2}$ |  | 12. $\ln \sqrt[3]{m^{2} n}$ |


| Name: |  |  | Date: |
| :---: | :---: | :---: | :---: |
| Topic: |  |  | Class: |
| Main Ideas/Questions | Notes/Examples |  |  |
| What is "e"? | - $e$ is an irrational $\qquad$ number with an approximate value of $\qquad$ 2.71828... . <br> - $e$ often occurs as the base of exponential and logarithmic functions that describe real-world scenarios. |  |  |
| Base "e" Exponentlal functions | - Exponential functions with base $e$ are called natural base $\qquad$ exponential functions. <br> - Example: $\qquad$ $f(x)=e^{x}$ |  |  |
| Base "e* Logarithmic Functions | - Logarithmic functions with base $e$ are called $\qquad$ natural logarithms $\qquad$ $\qquad$ <br> - Example: $f(x)=\log _{e} x$. This is often abbreivated as $f(x)=\ln x$. |  |  |
| Converting Between Forms | Witie each equation in logarithmic form. |  |  |
|  | $\begin{aligned} & \text { 1. } e^{x}=24 \\ & \log _{e} 24=x \\ & \ln 24=x \end{aligned}$ | $\begin{array}{r} \text { 2. } \begin{array}{r} e^{9}=x \\ \log _{e} x \end{array}=9 \\ \ln x=9 \end{array}$ | $\begin{array}{r} \text { 3. } e^{x+5}=72 \\ \log _{e} 72=x+5 \\ \ln 72=x+5 \end{array}$ |
|  | Wite each equation in exponential form. |  |  |
|  |  | $\text { 5. } \begin{aligned} & \ln 6=x \\ & e^{x}=6 \end{aligned}$ | $\text { 6. } \begin{aligned} & \ln _{e}(x-9)=32 \\ & e^{32}=x-9 \end{aligned}$ |
| Smplifying with Properties | Condense each expression into a single logartihm. |  |  |
|  | $\begin{gathered} \text { 7. } \ln 3+\ln 16 \\ \ln 3 \cdot 16 \\ \ln 48 \end{gathered}$ | $\begin{aligned} \text { 8. } & \ln 63-2 \cdot \ln 3 \\ & \ln \frac{63}{3^{2}} \\ & \ln \frac{63}{9}=\ln 7 \end{aligned}$ | $\text { 9. } \begin{aligned} & \frac{1}{3} \cdot \ln 64+2 \cdot \ln x \\ & \ln 64^{1 / 3} \cdot x^{2} \\ & \ln 4 x^{2} \end{aligned}$ |
|  | Expand each logarithm. |  |  |
|  | $\begin{aligned} & \text { 10. } \ln 5 x \\ & \ln 5+\ln x \end{aligned}$ | $\begin{aligned} & \text { 11. } \ln \left(\frac{a^{3}}{b}\right)^{2}=\ln \frac{a b}{b^{2}} \\ & 6 \cdot \ln a-2 \cdot \ln b \end{aligned}$ | $\begin{aligned} & \text { 12. } \ln \sqrt[3]{m^{2} n} \\ & \ln m^{2 / 3} n^{1 / 3} \\ & \frac{2}{3} \cdot \ln m+\frac{1}{3} \ln n \end{aligned}$ |


| Solving Equations | Solve each equation below. Check for extraneous solutions. |  |
| :---: | :---: | :---: |
|  | 13. $\ln (4 x-27)=\ln (15-2 x)$ | 14. $2 \cdot \ln k=\ln (2 k+15)$ |
|  | 15. $\ln 72-\ln 4=\ln 6+\ln (a-2)$ | 16. $2 \cdot \ln (m+4)=\ln 4$ |
|  | 17. $\ln 8 x=2$ | 18. $\ln x-\ln 9=7$ |
|  | 19. $e^{x}=57$ | 20. $e^{y+3}-6=24$ |
|  | 21. $5 e^{4 n}=95$ | 22. $2 e^{c-9}+3=87$ |


| Solving Equations | Solve each equation below. Check for extraneous solutions. |  |
| :---: | :---: | :---: |
|  | 13. $\begin{aligned} \ln (4 x-27) & =\ln (15-2 x) \\ 4 x-27 & =15-2 x \\ 6 x & =42 \\ x & =7 \end{aligned}$ | $\text { 14. } 2 \cdot \ln k=\ln (2 k+15) ~ 子 \begin{aligned} & k^{2}=2 k+15 \\ & k^{2}-2 k-15=0 \\ & (k-5)(k+3)=0 \\ & k=5 \quad k>3 \end{aligned}$ |
|  | $\text { 15. } \begin{aligned} \ln 72-\ln 4 & =\ln 6+\ln (a-2) \\ \frac{72}{4} & =6(a-2) \\ 18 & =6 a-12 \\ 30 & =6 a \\ 5 & =a \end{aligned}$ | 16. 2 $\begin{aligned} & 2 \cdot \ln (m+4)=\ln 4 \\ & (m+4)^{2}=4 \\ & m^{2}+8 m+16=4 \\ & m^{2}+8 m+12=0 \\ & (m+6)(m+2)=0 \\ & m y<m=-2 \end{aligned}$ |
|  | $\begin{aligned} & \text { 17. } \ln e^{8 x}=2 \\ & e^{2}=8 x \\ & 7.3891=8 x \\ & 0.9236=x \end{aligned}$ | 18. $\begin{aligned} & 3 \ln x-\ln 9=7 \\ & \ln \frac{x}{9}=7 \\ & e^{7}=\frac{x}{9} \\ & 1096 \cdot 6332=\frac{x}{9} \\ & 9869.6984=x \end{aligned}$ |
|  | $\begin{gathered} 19 . e^{x}=57 \\ \log _{e} 57=x \\ \ln 57=x \\ 4.0431=x \end{gathered}$ | $\begin{gathered} \text { 20. } e^{y+3}-6=24 \\ e^{y+3}=30 \\ \log _{e} 30=y+3 \\ \ln 30=y+3 \\ 3.4012=y+3 \\ 0.4012=y \end{gathered}$ |
|  | $\begin{aligned} & 21.5 e^{4 n}=95 \\ & e^{4 n}=19 \\ & \log _{e} 19=4 n \\ & \ln 19=4 n \\ & 2.9444=4 n \\ & 0.7361=n \end{aligned}$ | 22. $\begin{gathered} 2 e^{c-9}+3=87 \\ 2 e^{c-9}=84 \\ e^{c-9}=42 \\ \log _{e} 42=c-9 \\ \ln 42=c-9 \\ 3.7377=c-9 \\ 12.7377=c \end{gathered}$ |



Name: $\qquad$ Unit 7: Exponential \& Logarithmic Functions


Date: $\qquad$ Bell: $\qquad$ Homework 9: Base e and Natural Logarithms
** This is a 2-page document: **
Directions: Write each equation in logarithmic form.

| 1. $\begin{aligned} & e^{3}=x \\ & 3=\ln X \end{aligned}$ | $\text { 2. } \begin{aligned} e^{x} & =36 \\ x & =\ln 36 \end{aligned}$ | $\text { 3. } \begin{aligned} e^{x-9} & =74 \\ x-9 & =\ln 74 \end{aligned}$ |
| :---: | :---: | :---: |
| Directions: Write each equation in exponential form. |  |  |
| $\begin{aligned} & \text { 4. } \ln 53=x \\ & 53=e^{x} \end{aligned}$ | $\begin{aligned} & \text { 5. } \ln x=18 \\ & x=e^{18} \end{aligned}$ | $\begin{aligned} 6 . \ln 87 & =x+4 \\ 87 & =e^{x+4} \end{aligned}$ |
| Directions: Condense each expression as a single logarithm. |  |  |
| $\begin{array}{r} \text { 7. } \ln 4+\ln 3 x \\ \ln 12 x \end{array}$ | $\text { 8. } \frac{1}{2} \cdot \ln 256-3 \cdot \ln 2 ~\left(\begin{array}{c} \ln \frac{256^{6 / 2}}{2^{3}} \\ \ln 2 \end{array}\right.$ | 9. $7 \cdot \ln a-4 \cdot \ln b$ $\ln \frac{a^{7}}{b^{4}}$ |
| Directions: Expand each logarithmic expression. |  |  |
| $\begin{aligned} & \text { 10. } \ln \left(2 m^{8}\right) \\ & \ln 2+8 \ln m \end{aligned}$ | $\begin{aligned} & \text { 11. } \ln \left(\frac{m^{5}}{n^{2}}\right)^{3}=\ln \frac{m^{15}}{n^{6}} \\ & 15 \cdot \ln m-6 \cdot \ln n \end{aligned}$ | $\begin{aligned} & \text { 12. } \ln \sqrt{r^{8} s^{5}}=\ln r^{4} s^{5 / 2} \\ & 4 \cdot \ln r+\frac{5}{2} \cdot \ln s \end{aligned}$ |
| Directions: Solve each equation. Be sure to check for extraneous solutions. |  |  |
| 13. $\begin{gathered} \ln (9 x-7)=\ln (5 x+33) \\ 9 x-7=5 x+33 \\ 4 x=40 \\ x=10 \end{gathered}$ | $\text { 14. } \begin{array}{r} \ln \left(2 x^{2}\right. \\ 2 x^{2} \\ \\ x \\ x= \\ x \end{array}$ | $\begin{aligned} & =\ln \left(x^{2}+34\right) \\ & =x^{2}+34 \\ & =49 \\ & 49=0 \\ & (x-7)=0 \\ & x=7 \end{aligned}$ |


| 15. $\ln 60-\ln 4=\ln \left(x^{2}+2 x\right)$ | 16. $\ln 8+\ln (n-9)=5 \cdot \ln 2$ |
| :--- | :--- |
| 17. $\ln (4 w+9)=5$ | 18. $\ln k-\ln 14=2$ |
| 19. $e^{x}=21$ | 20. $-2 e^{c}+14=-6$ |
|  |  |
| 21. $e^{y-1}-27=54$ | 24. $3 e^{4 m-7}-8=106$ |


| $\text { 15. } \begin{array}{rl} \ln 60-\ln 4=\ln \left(x^{2}+2 x\right) \\ \frac{60}{4} & =x^{2}+2 x \\ 0 & =x^{2}+2 x-15 \\ 0 & =(x+5)(x-3) \\ x=-5 & x=3 \end{array}$ | 16. $\ln 8+\ln (n-9)=5 \cdot \ln 2$ $\begin{aligned} 8(n-9) & =2^{5} \\ 8 n-72 & =32 \\ 8 n & =104 \\ n & =13 \end{aligned}$ |
| :---: | :---: |
| $\text { 17. } \begin{aligned} \ln (4 w+9) & =5 \\ 4 w+9 & =e^{5} \\ 4 w+9 & =148.4132 \\ 4 w & =139.4132 \\ w & =34.8533 \end{aligned}$ | 18. $\ln$ $\begin{aligned} & n k-\ln 14=2 \\ & \frac{K}{14}=e^{2} \\ & \frac{K}{14}=7.3891 \\ & K=103.4468 \end{aligned}$ |
| 19. $\begin{aligned} & e^{x}=21 \\ & x=\ln 21 \\ & x=3.0445 \end{aligned}$ | 20. $\begin{aligned} -2 e^{c}+14 & =-6 \\ -2 e^{c} & =-20 \\ e^{c} & =10 \\ c & =\ln 10 \\ c & =2.3026 \end{aligned}$ |
| 21. $\begin{aligned} & e^{y-1}-27=54 \\ & e^{y-1}=81 \\ & y-1=\ln 81 \\ & y-1=4.3944 \\ & y=5.3944 \end{aligned}$ | 22. $\begin{aligned} & 4 e^{3 k}+1=85 \\ & 4 e^{3 k}=84 \\ & e^{3 k}=21 \\ & 3 k=\ln 21 \\ & 3 k=3.0445 \\ & k=1.0148 \end{aligned}$ |
| 23. $\begin{aligned} 3 . e^{5-2 p}+2 & =4 \\ e^{5-2 p} & =2 \\ 5-2 p & =\ln 2 \\ 5-2 p & =0.6931 \\ -2 p & =-4.3069 \\ p & =2.1534 \end{aligned}$ | 24. 3 $\begin{aligned} 3 e^{i m-7}-8 & =106 \\ 3 e^{4 m-7} & =114 \\ e^{4 m-7} & =38 \\ 4 m-7 & =\ln 38 \\ 4 m-7 & =3.6376 \\ 4 m & =10.6376 \\ m & =2.6594 \end{aligned}$ |

