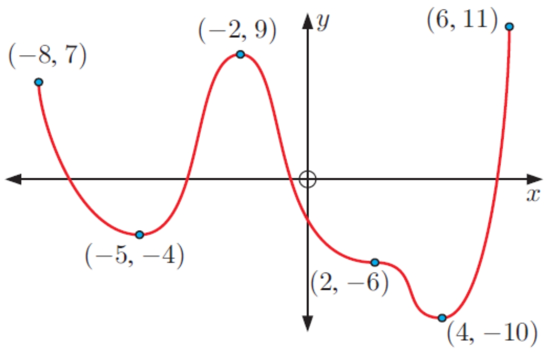
21B p. 593 2,5,7,8,10 21C.1 p. 596 #1,3,5 21.C.2 p.598 #6 and 9

Max and Min and Rate of Change Homework:

1. Consider the graph of $y = f(x)$ on the domain $-8 \leqslant x \leqslant 6$.



- **a** How many stationary points does the graph have?
- **b** Write down the coordinates of the:
  - **i** local maximum
  - **ii** horizontal inflection.
- **c** Find the **i** greatest **ii** least value of $f(x)$ on $-8 \leqslant x \leqslant 6$.
- **d** Find the greatest value of $f(x)$ on $-8 \leqslant x \leqslant 4$.
- **e** Find the least value of $f(x)$ on $-5 \leqslant x \leqslant 2$.

2. For each of the following functions:

- **i** find $f'(x)$
- **ii** draw a sign diagram for $f'(x)$
- **iii** determine the position and nature of all stationary points
- **iv** sketch $y = f(x)$, showing all key features.

- **a** $f(x) = x^2 - 2$
- **b** $f(x) = x^3 + 1$
- **d** $f(x) = x^4 - 2x^2$
- **e** $f(x) = x^3 - 6x^2 + 12x + 1$
- **g** $f(x) = 2x + \frac{1}{x^2}$
- **h** $f(x) = -x - \frac{9}{x}$

3. $f(x) = 2x^3 + ax^2 - 24x + 1$ has a local maximum at $x = -4$. Find a.

4. $f(x) = x^3 + ax + b$ has a stationary point at $(-2, 3)$. Find:

- **a** a and b
- **b** $f'(x)$
- **c** the position and nature of all stationary points.

5. A manufacturing company makes door hinges. They have a standing order filled by producing 50 each hour, but production of more than 150 per hour is useless as they will not sell. The cost function for making x hinges per hour is:

$$C(x) = 0.0007x^3 - 0.1796x^2 + 14.663x + 160 \text{ dollars where } 50 \leqslant x \leqslant 150.$$

- **a** Find $C'(x)$.
- **b** Find the values of x for which $C'(x) = 0$.
- **c** Find the minimum and maximum hourly costs, and the production levels when each occurs.

6. Find:

a $\dfrac{dM}{dt}$ if $M = t^3 - 3t^2 + 1$ **b** $\dfrac{dR}{dt}$ if $R = (2t+1)^2$

7. The number of bacteria in a dish is modelled by $B(t) = 0.3t^2 + 30t + 150$ thousand, where t is in days, and $0 \leqslant t \leqslant 10$.

a Find $B'(t)$ and state its meaning.

b Find $B'(3)$ and state its meaning.

c How do we know that $B(t)$ is increasing over the first 10 days?

8. When a ball is thrown, its height above the ground is given by $s(t) = 1.2 + 28.1t - 4.9t^2$ metres, where t is the time in seconds.

a From what distance above the ground was the ball released?

b Find $s'(t)$ and explain what it means.

c Find t when $s'(t) = 0$. What is the significance of this result?

d What is the maximum height reached by the ball?

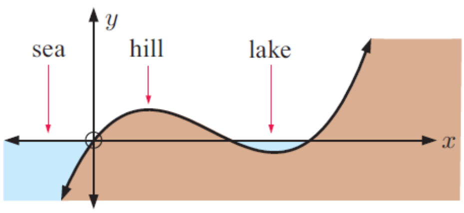
e Find the ball's speed:

i when released **ii** at $t = 2$ s **iii** at $t = 5$ s.

State the significance of the sign of the derivative $s'(t)$ for each of these values.

f How long will it take for the ball to hit the ground?

9.



Alongside is a land and sea profile where the x-axis is sea level.

The function $y = \frac{1}{10}x(x-2)(x-3)$ km gives the height of the land or sea bed relative to sea level.

a Find where the lake is located relative to the shore line of the sea.

b Find $\dfrac{dy}{dx}$ and interpret its value when $x = \frac{1}{2}$ and when $x = 1\frac{1}{2}$ km.

c Find the point at which the lake floor is level, and the depth at this point.

10. The cost of producing x items is given by $C(x) = 0.0002x^3 + 0.04x^2 + 10x + 3000$ dollars. If each item sells for \$70, find:

a the revenue function $R(x)$ **b** the profit function $P(x)$

c $P'(x)$ **d** $P'(120)$, and explain the significance of this result.