Logarithmic Functions

Remember: The inverse of a function , written f^{-1}, is a function such that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

Ex 1: Show that $f(x) = 3x + 1$ and $g(x) = 1/3(x - 1)$ are inverses.

How do you find the inverse of a function?

Ex 2) Find the inverse of f(x) = $\frac{2}{3+x}$

Silent Board Game:

8	1/2	32	1	16	4	3	64	2	0	.25	-1
3				4						-2	

$\sqrt{2}$	.2	1/8

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
What is a **LOGARITHM?**	**A logarithm (log) is another way of writing exponents.** **Logarithmic Form** $\log_b a = x$ ➡ **Exponential Form** Read as "**log base b of a equals x.**"	
Converting **LOG ➪ EXP**	**Directions:** Write each equation in **exponential form**.	
	1. $\log_3 9 = 2$	**2.** $\log_6 216 = 3$
	3. $\log_7 1 = 0$	**4.** $\log_2 16 = 4$
	5. $\log_4 \frac{1}{16} = -2$	**6.** $\log_9 27 = \frac{3}{2}$
Converting **EXP ➪ LOG**	**Directions:** Write each equation in **logarithmic form**.	
	7. $14^2 = 196$	**8.** $3^4 = 81$
	9. $12^1 = 12$	**10.** $36^{\frac{1}{2}} = 6$
	11. $2^{-3} = \frac{1}{8}$	**12.** $8^{\frac{4}{3}} = 16$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
What is a **LOGARITHM?**	**A logarithm (log) is another way of writing exponents.** **Logarithmic Form** $\log_b a = x$ ➡ **Exponential Form** $b^x = a$ ↳ Read as "**log base b of a equals x.**"
Converting **LOG ➲ EXP**	**Directions:** Write each equation in **exponential form.**
	1. $\log_3 9 = 2$ $3^2 = 9$
	2. $\log_6 216 = 3$ $6^3 = 216$
	3. $\log_7 1 = 0$ $7^0 = 1$
	4. $\log_2 16 = 4$ $2^4 = 16$
	5. $\log_4 \frac{1}{16} = -2$ $4^{-2} = \frac{1}{16}$
	6. $\log_9 27 = \frac{3}{2}$ $9^{3/2} = 27$
Converting **EXP ➲ LOG**	**Directions:** Write each equation in **logarithmic form.**
	7. $14^2 = 196$ $\log_{14} 196 = 2$
	8. $3^4 = 81$ $\log_3 81 = 4$
	9. $12^1 = 12$ $\log_{12} 12 = 1$
	10. $36^{\frac{1}{2}} = 6$ $\log_{36} 6 = \frac{1}{2}$
	11. $2^{-3} = \frac{1}{8}$ $\log_2 \frac{1}{8} = -3$
	12. $8^{\frac{4}{3}} = 16$ $\log_8 16 = \frac{4}{3}$

COMMON LOGARITHM

A logarithm with base 10 is called a **common logarithm** and can be written without the base.

$\log_{10} x \longrightarrow$

EVALUATING LOGARITHMS

Directions: Use your knowledge of exponents to evaluate the following logarithms.

13. $\log_7 49$	**14.** $\log_3 27$
15. $\log 100$	**16.** $\log_{12} 1$
17. $\log_2 64$	**18.** $\log_3 243$
19. $\log_9 \frac{1}{81}$	**20.** $\log_{64} 4$

CHANGE OF BASE FORMULA

Some logarithms are not as easy to evalute as those above, and will require the **change of base formula**.

$\log_b a =$

Choose BASE 10 because there is a calculator button for it! →

Directions: Evalute each log using the change of base formula.

21. $\log_{16} 64$	**22.** $\log_8 32$
23. $\log_2 54$	**24.** $\log_{10} 294$
25. $\log_4 136$	**26.** $\log_6 \frac{1}{36}$

COMMON LOGARITHM

A logarithm with base 10 is called a **common logarithm** and can be written without the base. $\log_{10} x \longrightarrow \log x$

EVALUATING LOGARITHMS

Directions: Use your knowledge of exponents to evaluate the following logarithms.

13. $\log_7 49$

$7^x = 49$

$x = 2$

14. $\log_3 27$

$3^x = 27$

$x = 3$

15. $\log 100$

$10^x = 100$

$x = 2$

16. $\log_{12} 1$

$12^x = 1$

$x = 0$

17. $\log_2 64$

$2^x = 64$

$x = 6$

18. $\log_3 243$

$3^x = 243$

$x = 5$

19. $\log_9 \frac{1}{81}$

$9^x = \frac{1}{81}$ $x = -2$

20. $\log_{64} 4$

$64^x = 4$

$x = \frac{1}{3}$

CHANGE OF BASE FORMULA

Some logarithms are not as easy to evalute as those above, and will require the **change of base formula.**

$$\log_b a = \frac{\log a}{\log b}$$

Choose BASE 10 because there is a calculator button for it!

Directions: Evalute each log using the change of base formula.

21. $\log_{16} 64$

$\frac{\log 64}{\log 16} = 1.5$

22. $\log_8 32$

$\frac{\log 32}{\log 8} = 1.\overline{6}$

23. $\log_2 54$

$\frac{\log 54}{\log 2} = 5.7549$

24. $\log_{10} 294$

$\frac{\log 294}{\log 10} = 2.4683$

25. $\log_4 136$

$\frac{\log 136}{\log 4} = 3.5437$

26. $\log_6 \frac{1}{36}$

$\frac{\log \frac{1}{36}}{\log 6} = -2$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples		
Product Property $\log_b(m \cdot n) =$	**Condense into a single logarithm. Simplify if possible.**		
	1. $\log_2 7 + \log_2 4$	**2.** $\log 25 + \log 4$	**3.** $\log_4 2x + \log_4 4x^2$
	Expand using the product property.		
	4. $\log 6$	**5.** $\log_7 45$	**6.** $\log_2 (5x)$
Quotient Property $\log_b \left(\frac{m}{n}\right) =$	**Condense into a single logarithm. Simplify if possible.**		
	7. $\log_3 24 - \log_3 8$	**8.** $\log_2 15 - \log_2 15$	**9.** $\log_4 x^9 - \log_4 x^2$
	Expand using the quotient property.		
	10. $\log_8 4$	**11.** $\log_5 \frac{1}{3}$	**12.** $\log \left(\frac{m}{7}\right)$
Power Property $\log_b m^n =$	**Condense into a single logarithm. Simplify if possible.**		
	13. $5 \cdot \log_4 2$	**14.** $7 \cdot \log_2 x$	**15.** $\frac{1}{3} \cdot \log 8$
	Expand using the power property. Simplify if possible.		
	16. $\log_2 8^7$	**17.** $3 \cdot \log 4^{x-1}$	**18.** $\log_7 \sqrt{w}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples		
Product Property $\log_b(m \cdot n) =$ $\log_b m + \log_b n$ Answers may vary. →	**Condense into a single logarithm. Simplify if possible.**		
	1. $\log_2 7 + \log_2 4$ $\log_2 7 \cdot 4$ $\log_2 28$	**2.** $\log 25 + \log 4$ $\log 25 \cdot 4$ $\log 100$	**3.** $\log_4 2x + \log_4 4x^2$ $\log_4 2x \cdot 4x^2$ $\log_4 8x^3$
	Expand using the product property.		
	4. $\log 6$ $\log 2 \cdot 3$ $\log 2 + \log 3$	**5.** $\log_7 45$ $\log_7 5 \cdot 9$ $\log_7 5 + \log_7 9$	**6.** $\log_2 (5x)$ $\log_2 5 \cdot x$ $\log_2 + \log_2 x$
Quotient Property $\log_b\left(\frac{m}{n}\right) =$ $\log_b m - \log_b n$ Answers may vary! →	**Condense into a single logarithm. Simplify if possible.**		
	7. $\log_3 24 - \log_3 8$ $\log_3 \frac{24}{8}$ $\log_3 3$	**8.** $\log_2 15 - \log_2 15$ $\log_2 \frac{15}{15}$ $\log_2 1$	**9.** $\log_4 x^9 - \log_4 x^2$ $\log_4 \frac{x^9}{x^2}$ $\log_4 x^7$
	Expand using the quotient property.		
	10. $\log_8 4$ $\log_8 \frac{8}{2}$ $\log_8 8 - \log_8 2$	**11.** $\log_5 \frac{1}{3}$ $\log_5 \frac{15}{45}$ $\log_5 15 - \log_5 45$	**12.** $\log\left(\frac{m}{7}\right)$ $\log m - \log 7$
Power Property $\log_b m^n =$ $n \cdot \log_b m$	**Condense into a single logarithm. Simplify if possible.**		
	13. $5 \cdot \log_4 2$ $\log_4 2^5$ $\log_4 32$	**14.** $7 \cdot \log_2 x$ $\log_2 x^7$	**15.** $\frac{1}{3} \cdot \log 8$ $\log 8^{1/3}$ $\log \sqrt[3]{8}$
	Expand using the power property. Simplify if possible.		
	16. $\log_2 8^7$ $7 \cdot \log_2 8$	**17.** $3 \cdot \log 4^{x-1}$ $x-1 \cdot 3 \cdot \log 4$ $(3x-3) \cdot \log 4$	**18.** $\log_7 \sqrt{w}$ $\log_7 w^{1/2}$ $\frac{1}{2} \cdot \log_7 w$

Name: _____ **Unit 7:** Exponential & Logarithmic Functions

Date: _____ **Bell:** _____ **Homework 3:** Intro to Logarithms

Directions: Write each equation in exponential form.		
1. $\log_2 128 = 7$	**2.** $\log_8 64 = 2$	**3.** $\log_3 \frac{1}{27} = -3$
Directions: Write each equation in logarithmic form.		
4. $4^4 = 256$	**5.** $8^3 = 512$	**6.** $27^{\frac{2}{3}} = 9$
Directions: Evaluate each logarithm. Use the change of base formula when necessary.		
7. $\log_6 36$	**8.** $\log_2 32$	**9.** $\log_4 64$
10. $\log_3 81$	**11.** $\log_{100} 10$	**12.** $\log_7 \frac{1}{7}$
13. $\log_{18} 1$	**14.** $\log_2 \frac{1}{16}$	**15.** $\log 1000$
16. $\log_{16} 8$	**17.** $\log_{243} 27$	**18.** $\log_3 92$
19. $\log_7 35$	**20.** $\log_2 260$	**21.** $\log_5 38$

Name: _____ **Unit 7:** Exponential & Logarithmic Functions

Date: _____ **Bell:** _____ **Homework 3:** Intro to Logarithms

Directions: Write each equation in exponential form.

1. $\log_2 128 = 7$

$2^7 = 128$

2. $\log_8 64 = 2$

$8^2 = 64$

3. $\log_3 \frac{1}{27} = -3$

$3^{-3} = \frac{1}{27}$

Directions: Write each equation in logarithmic form.

4. $4^4 = 256$

$\log_4 256 = 4$

5. $8^3 = 512$

$\log_8 512 = 3$

6. $27^{\frac{2}{3}} = 9$

$\log_{27} 9 = \frac{2}{3}$

Directions: Evaluate each logarithm. Use the change of base formula when necessary.

7. $\log_6 36$

$6^x = 36$

$x = 2$

8. $\log_2 32$

$2^x = 32$

$x = 5$

9. $\log_4 64$

$4^x = 64$

$x = 3$

10. $\log_{-3} 81$

$3^x = 81$

$x = 4$

11. $\log_{100} 10$

$100^x = 10$

$x = 1/2$

12. $\log_7 \frac{1}{7}$

$7^x = \frac{1}{7}$

$x = -1$

13. $\log_{18} 1$

$18^x = 1$

$x = 0$

14. $\log_2 \frac{1}{16}$

$2^x = \frac{1}{16}$

$x = -4$

15. $\log 1000$

$10^x = 1000$

$x = 3$

16. $\log_{16} 8$

$\frac{\log 8}{\log 16} = 0.75$

17. $\log_{243} 27$

$\frac{\log 27}{\log 243} = 0.6$

18. $\log_3 92$

$\frac{\log 92}{\log 3} = 4.1159$

19. $\log_7 35$

$\frac{\log 35}{\log 7} = 1.8271$

20. $\log_2 260$

$\frac{\log 260}{\log 2} = 8.0024$

21. $\log_5 38$

$\frac{\log 38}{\log 5} = 2.2602$

Putting it All Together!

CONDENSING LOGS

Directions: Rewrite as a single logarithm. Simplify if possible.

19. $2 \cdot \log 6 - \log 9$

20. $4 \cdot \log_4 a + 2 \cdot \log_4 b$

21. $7 \cdot \log_4 u - 3 \cdot \log_4 v^2$

22. $\log_2 15 + \log_2 4 - \log_2 6$

23. $\log_3 4 + \log_3 y + \frac{1}{2} \cdot \log_3 49$

24. $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

25. $3 \cdot \log_2 4 - \log_2 32$

26. $2 \cdot \log 6 - \frac{1}{4} \cdot \log 16 + \log 3$

EXPANDING LOGS

Directions: Expand each logarithm.

27. $\log_6 (xyz^4)$

28. $\log_4 \left(\frac{a^9}{b}\right)$

29. $\log_7 (q^4 r^2)^2$

30. $\log_2 \left(\frac{y}{z^5}\right)^2$

31. $\log \sqrt{7x^3}$

32. $\log_3 \sqrt[4]{m^5 n^2}$

Putting it All Together!

CONDENSING LOGS

Directions: Rewrite as a single logarithm. Simplify if possible.

19. $2 \cdot \log 6 - \log 9$

$\log \frac{6^2}{9} = \log \frac{36}{9} = \boxed{\log 4}$

20. $4 \cdot \log_4 a + 2 \cdot \log_4 b$

$\log_4 a^4 + \log_4 b^2$

$\boxed{\log_4 a^4 b^2}$

21. $7 \cdot \log_4 u - 3 \cdot \log_4 v^2$

$\log_4 u^7 - \log_4 v^6$

$\boxed{\log_4 \frac{u^7}{v^6}}$

22. $\log_2 15 + \log_2 4 - \log_2 6$

$\log_2 \frac{15 \cdot 4}{6}$

$\boxed{\log_2 10}$

23. $\log_3 4 + \log_3 y + \frac{1}{2} \cdot \log_3 49$

$\log_3 4 \cdot y \cdot 49^{1/2}$

$\log_3 4 \cdot y \cdot 7 = \boxed{\log_3 28y}$

24. $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

$\log_5 \frac{8^{1/3} \cdot 27^{1/3}}{3} - \log_5 \frac{2 \cdot 3}{3}$

$= \boxed{\log_5 2}$

25. $3 \cdot \log_2 4 - \log_2 32$

$\log_2 \frac{4^3}{32} = \log_2 \frac{64}{32}$

$= \boxed{\log_2 2}$

26. $2 \cdot \log 6 - \frac{1}{4} \cdot \log 16 + \log 3$

$\log \frac{6^2}{16^{1/4} \cdot 3} = \log \frac{36}{6} = \boxed{\log 6}$

EXPANDING LOGS

Directions: Expand each logarithm.

27. $\log_6 (xyz^4)$

$\log_6 x + \log_6 y + \log_6 z^4$

$\boxed{\log_6 x + \log_6 y + 4 \log_6 z}$

28. $\log_4 \left(\frac{a^9}{b}\right)$

$\log_4 a^9 - \log_4 b$

$\boxed{9 \cdot \log_4 a - \log_4 b}$

29. $\log_7 (q^4 r^2)^2$

$2(\log_7 q^4 + \log_7 r^2)$

$2(4 \cdot \log_7 q + 2 \log_7 r) = \boxed{8 \log_7 q + 4 \log_7 r}$

30. $\log_2 \left(\frac{y}{z^5}\right)^2 = 2(\log_2 y - \log_2 z^5)$

$= 2(\log_2 y - 5 \log_2 z)$

$= \boxed{2 \log_2 y - 10 \log_2 z}$

31. $\log \sqrt{7x^3} = \frac{1}{2}(\log 7 + \log x^3)$

$= \frac{1}{2}(\log 7 + 3 \log x)$

$= \boxed{\frac{1}{2} \log 7 + \frac{3}{2} \log x}$

32. $\log_3 \sqrt[4]{m^5 n^2} = \frac{1}{4}(\log_3 m^5 + \log_3 n^2)$

$= \frac{1}{4}(5 \log_3 m + 2 \log_3 n)$

$= \boxed{\frac{5}{4} \log_3 m + \frac{1}{2} \log_3 n}$

PROPERTIES OF LOGARITHMS

Name	Rule(s)	Exam
BASIC LOGARITHMS	$\log_b b =$; $\log_b 1 =$	**Simp** $\log_{14} 14 =$
PRODUCT RULE	$\log_b (m \cdot n) =$	**Cond** $\log_5 6 +$
QUOTIENT RULE	$\log_b \left(\frac{m}{n}\right) =$	**Cond** $\log_4 84 -$
POWER RULE	$\log_b m^n =$	**Cond** $2 \cdot \log$
CHANGE OF BASE FORMULA	$\log_b a =$	**Usin** $\log_7 32 =$
REMEMBER: BASE 10 LOGS ARE COMMON LOGS AND WRITTEN		

PROPERTIES OF LOGARITHMS

GRAPHIC ORGANIZER

Name	Rule(s)	Example 1	Example 2
BASIC LOGARITHMS	$\log_b b = 1$; $\log_b 1 = 0$	**Simplify:** $\log_{14} 14 = 1$	**Simplify:** $\log_3 1 = 0$
PRODUCT RULE	$\log_b (m \cdot n) = \log_b m + \log_b n$	**Condense:** $\log_5 6 + \log_5 7 =$ $\log_5 6 \cdot 7$ $\log_5 42$	**Expand:** $\log_2 63 =$ $\log_2 7 \cdot 9$ $\log_2 7 + \log_2 9$
QUOTIENT RULE	$\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$	**Condense:** $\log_4 84 - \log_4 12 =$ $\log_4 \frac{84}{12} = \log_4 7$	**Expand:** $\log 9 =$ $\log \frac{81}{9} = \log 81 - \log 9$
POWER RULE	$\log_b m^n = n \cdot \log_b m$	**Condense:** $2 \cdot \log_3 8 =$ $\log_3 8^2 = \log_3 64$	**Expand:** $\log_2 6^{x-1} =$ $(x-1) \cdot \log_2 6$
CHANGE OF BASE FORMULA	$\log_b a = \frac{\log a}{\log b}$	**Using a common base, evaluate the expression below.** $\log_7 32 = \frac{\log 32}{\log 7} = 1.7810$	
REMEMBER: BASE 10 LOGS ARE COMMON LOGS AND WRITTEN WITHOUT A BASE! **(log x)**			

Name: _____ **Unit 7:** Exponential & Logarithmic Functions

Date: _____ **Bell:** _____ **Homework 4:** Properties of Logarithms

**** This is a 2-page document! ****

Directions: Complete each rule.

PRODUCT RULE	QUOTIENT RULE	POWER RULE
$\log_b(m \cdot n) =$	$\log_b\left(\frac{m}{n}\right) =$	$\log_b m^n =$
$\log_b m + \log_b n$	$\log_b m - \log_b n$	$n \cdot \log_b m$

Directions: Condense each expression into a single logarithm. Simplify if possible.

1. $\log_7 9 + \log_7 4$

$\log_7 9 \cdot 4 = \boxed{\log_7 36}$

2. $\log_2 80 - \log_2 5$

$\log_2 \frac{80}{5} = \boxed{\log_2 16}$

3. $\frac{1}{2} \cdot \log_3 81$

$\log_3 81^{1/2} = \log_3 \sqrt{81}$

$= \boxed{\log_3 9}$

4. $3 \cdot \log_4 8 - 5 \cdot \log_4 2$

$\log_4 \frac{8^3}{2^5} = \log_4 \frac{512}{32}$

$= \boxed{\log_4 16}$

5. $7 \cdot \log_5 x + 3 \cdot \log_5 y^4$

$\log_5 x^7 \cdot y^{12}$

$= \boxed{\log_5 x^7 y^{12}}$

6. $\log_3 a^7 + (\log_3 a^4 - 2 \cdot \log_3 b)$

$\log_3 a^7 \cdot \frac{a^4}{b^2} = \boxed{\log_3 \frac{a^{11}}{b^2}}$

7. $\log_4 x^7 - \log_4 x^2 + 4 \cdot \log_4 x^3$

$\log_4 \frac{x^7}{x^2} \cdot x^{12} = \log_4 \frac{x^{19}}{x^2}$

$= \boxed{\log_4 x^{17}}$

8. $\frac{1}{2}(\log_6 45 - \log_6 5) + \log_6 12$

$\log_6 \left(\frac{45}{5}\right)^{1/2} \cdot 12$

$= \log_6 9^{1/2} \cdot 12$

$= \log_6 3 \cdot 12 = \boxed{\log_6 36}$

Directions: Condense, then use the **change of base formula** to evaluate the logarithm.

9. $\log_9 35 - \log_9 7$

$\log_9 \frac{35}{7}$

$\log_9 5 \rightarrow \frac{\log 5}{\log 9} = 0.7325$

10. $2 \cdot \log_3 8 - 4 \cdot \log_3 2$

$\log_3 \frac{8^2}{2^4}$

$\log_3 \frac{64}{16}$

$\log_3 4 \rightarrow \frac{\log 4}{\log 3} = 1.2619$

11. $\frac{1}{3} \cdot \log_4 8 + \log_4 15$

$\log_4 8^{1/3} \cdot 15$

$\log_4 2 \cdot 15$

$\log_4 30 \rightarrow \frac{\log 30}{\log 4} = 2.4534$

12. $\log_2 27 + \log_2 4 - 2 \cdot \log_2 3$

$\log_2 \frac{27 \cdot 4}{3^2}$

$\log_2 \frac{108}{9}$

$\log_2 12 \rightarrow \frac{\log 12}{\log 2} = 3.5850$

Directions: Expand each expression.

13. $\log_8 (mn^2)$

$\log_8 m + 2\log_8 n$

14. $\log \left(\frac{x^9}{y^4}\right)$

$9 \log x - 4 \log y$

15. $\log_2 (a^5 b^2)^3$

$\log_2 (a^{15} b^6)$

$15 \log_2 a + 6 \log_2 b$

16. $\log \left(\frac{p^4}{q^7}\right)^2$

$\log \frac{p^8}{q^{14}}$

$8 \log p - 14 \log q$

17. $\log_5 \sqrt{a^7 b}$

$\log_5 a^{7/2} b^{1/2}$

$\frac{7}{2} \log_5 a + \frac{1}{2} \log_5 b$

18. $\log_4 \sqrt[3]{c^2 d^{15}}$

$\log_4 c^{2/3} d^5$

$\frac{2}{3} \log_4 c + 5 \log_4 d$