

Implication and Equivalence HW:

- In the following implicative statements, state the antecedent and the consequent.
 - If I miss the bus, then I will walk to school.
 - If the temperature is low enough, then the lake will freeze.
 - If $x > 20$, then $x > 10$.
 - If you jump all 8 hurdles, then you may win the race.
- For the following propositions, write down the implicative statement $p \Rightarrow q$:
 - p : The sun is shining, q : I will go swimming
 - p : x is a multiple of 6, q : x is even
 - p : There are eggs in the fridge, q : Jan will bake a cake.
- For the following propositions p and q :
 - write down the equivalence $p \Leftrightarrow q$
 - state whether the equivalence is true or false.
 - p : Rome is the capital of Italy, q : Paris is the capital of France
 - p : $2x + 3 = 10$ is an expression, q : $2x + 3$ is an expression
 - p : Cows have nine legs, q : Horses have five heads.

- Consider the propositions p : It is raining and q : There are puddles forming. Write the following statements in symbols:

- If it is raining then puddles are forming.
- If puddles are forming then it is raining.
- Puddles are not forming.
- It is not raining.
- If it is not raining, then puddles are not forming.
- If it is raining, then puddles are not forming.
- If there are no puddles, then it is raining.
- It is raining if and only if there are puddles forming.



- Construct truth tables for:

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| a $p \Rightarrow \neg q$ | b $\neg q \Rightarrow \neg p$ | c $(p \wedge q) \Rightarrow p$ | d $q \wedge (p \Rightarrow q)$ |
| e $p \Leftrightarrow \neg q$ | f $(p \Leftrightarrow q) \wedge \neg p$ | g $p \Rightarrow (p \wedge \neg q)$ | h $(p \Rightarrow q) \Rightarrow \neg p$ |

- By examining truth tables, show that:

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| a $p \vee q = \neg(p \Leftrightarrow q)$ | b $\neg p \Rightarrow q = p \vee q$ |
| c $q \Rightarrow (p \vee q) = \neg(p \wedge q)$ | d $p \Leftrightarrow q = (p \wedge q) \vee (\neg p \wedge \neg q)$ |

- Which of these forms are logically equivalent to the negation of $q \Rightarrow p$?

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| A $p \Rightarrow q$ | B $\neg q \Rightarrow p$ | C $q \Rightarrow \neg p$ | D $\neg(\neg p \Rightarrow \neg q)$ |
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- Determine whether the following are logical contradictions, tautologies, or neither:

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| a $p \Rightarrow (\neg p \wedge q)$ | b $(p \wedge q) \Rightarrow (p \vee q)$ | c $(p \Rightarrow \neg q) \vee (\neg p \Rightarrow q)$ |
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