ALGEBRA 2
2017 Spring Final Review

Name Key Block ____
Date: ____

I. Simplifying

A. Radicals

1. $\sqrt[3]{-24x^7m^5k^9}$

$= -2x^2mk^3\sqrt[3]{xm^2}$

2. $\sqrt[4]{81m^3y^{20}k^{17}}$

$= 3y^5k^4\sqrt[4]{m^3k}$

3. $\sqrt{6x^2}\cdot\sqrt{2x^5}$

$\sqrt{12x^7} = \sqrt{4\cdot3x^6\cdot x}$

$= 2x^3\sqrt{3x}$

B. Condense Logarithms

1. $\frac{1}{2}log_9m - 3log_9w$

$\log_9 m^{1/2} - \log_9 w^3$

$\log_9\left(\frac{\sqrt{m}}{w^3}\right)$

2. $4\log x + 5\log k - \frac{1}{3}\log w$

$\log x^4 + \log k^5 - \log w^{1/3}$

$\log\left(\frac{x^4k^5}{\sqrt[3]{w}}\right)$

3. $6log_7(x) + 3log_7(2x) - log_7(4)$

$\log_7 x^6 + \log_7(2x)^3 - \log_7(4)$

$\log_7\left(\frac{8x^9}{4}\right) = \log_7(2x^9)$

C. Laws of Exponents

1. $\frac{3x^2y^{-3}}{12x^6y^3} = \frac{1}{4x^4y^6}$

2. $\left(\frac{4x^5y^2}{16xy^4}\right)^3$ (cubed) $= \frac{x^{12}}{64y^6}$

$\left(\frac{x^4}{4y^2}\right)^3$

3. $\frac{2x^4y^{-4}}{8x^7y^3} = \frac{1}{4x^3y^7}$

D. Rational Expressions

1. $\frac{x^2+6x+5}{7x^2+4x-3}\cdot\frac{49x^2-9}{x^2+7x+10}$

$\frac{(x+5)(x+1)}{(7x-3)(x+1)}\cdot\frac{(7x+3)(7x-3)}{(x+5)(x+2)}$

-21, $7\times-3=-21$, $7-3=4$

$= \frac{7x+3}{x+2}$

2. $\frac{6x^2+47x-8}{x^4+6x^3-16x^2}\div\frac{36x^2-1}{4x^4-8x^3}$

$\frac{(6x-1)(x+8)}{x^2(x^2+6x-16)}\cdot\frac{(6x+1)(6x-1)}{4x^3(x-2)}$

-48, $48\times-1=-48$, $48-1=47$

$x^2(x+8)(x-2)$

work w/ factoring not answer.

$\frac{(6x-1)(x+8)}{x^2(x+8)(x-2)}\cdot\frac{4x^3(x-2)}{(6x+1)(6x-1)} = \frac{4x^3}{x^2(6x+1)}$

$= \frac{4x}{6x+1}$

3. $\frac{x-7}{3x-12}\cdot\frac{4x^2-16}{x^2-49}$

$3(x-4)$ $(x+7)(x-7)$

$4(x^2-4) = 4(x+2)(x-2)$

$\frac{4(x+2)(x-2)}{3(x-4)(x+7)}$

4. $\frac{x^2-25}{x^2+12x+35}$

$= (x+5)(x-5)$

$(x+7)(x+5)$

$\frac{x-5}{x+7}$

E. Composition Functions

If $f(x) = x^2 - 2x$, $g(x) = x - 4$, and $h(x) = \sqrt{x+2}$, find the following compositions:

1. $f(g(x)) = f(x-4)$

$= (x-4)^2 - 2(x-4)$

ok to leave like

$= x^2 - 8x + 16 - 2x + 8$

$= x^2 - 10x + 24$

2. $g(f(1)) = g(-1)$

$= -1 - 4$

$= -5$

3. $h(g(x)) = h(x-4)$

$= \sqrt{x-4+2}$

$= \sqrt{x-2}$

II. Solve. Check extraneous roots.

A. Exponential Equations

1. $5^{3x} = 25^{x-1}$

$3x = 2x - 2$

$x = -2$

2. $4^{-3v} = 64$

$-3v = 3$

$v = -1$

3. $8^{3x} = 4^{x+1}$

$(2^3)^{3x} = (2^2)^{x+1}$

$9x = 2x + 2$

$7x = 2$

$x = 2/7$

B. Logarithm Equations

1. $\log(3x - 2) = 2$

$10^2 = 3x - 2$

$3x = 102$

$x = 34$

2. $\log_3(x - 4) + \log_3(x + 4) = 2$

$\log_3(x^2 - 16) = 2$

$3^2 = x^2 - 16$

$25 = x^2$

$x = 5$ or $x \neq -5$ (extraneous)

3. $\log(x) + \log(x - 15) = 2$

$\log(x^2 - 15x) = 2$

$10^2 = x^2 - 15x$

$0 = x^2 - 15x - 100$

$0 = (x - 20)(x + 5)$

$x = 20$ or $x \neq -5$

C. Rational Equations

1. $\left(\frac{1}{x} + \frac{x+2}{x+5} = \frac{x+3}{x}\right) x(x+5)$

$x + 5 + x^2 + 2x = x^2 + 8x + 15$

$x^2 + 3x + 5 = x^2 + 8x + 15$

$3x + 5 = 8x + 15$

$-10 = 5x$

$x = -2$

2. $\left[\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{x^2-6x+8}\right] (x-2)(x-4)$

$(x-4)(x-2)$

$x(x-4) + (x-2) = 2$

$x^2 - 4x + x - 2 = 2$

$x^2 - 3x - 2 = 2$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x \neq 4$ or $x = -1$

3. $\left(\frac{x-1}{7x+6} = \frac{2}{x+2}\right) (7x+6)(x+2)$

$(x-1)(x+2) = 2(7x+6)$

$x^2 + x - 2 = 14x + 12$

$x^2 - 13x - 14 = 0$

$(x - 14)(x + 1) = 0$

$x = 14$ or $x = -1$

D. Radical Equations

1. $2\sqrt{3x - 1} + 3 = 11$

$2\sqrt{3x - 1} = 8$

$\sqrt{3x - 1} = 4$

$3x - 1 = 16$

$3x = 17$

$x = \frac{17}{3}$

2. $\sqrt{2x + 3} = x - 6$

$2x + 3 = x^2 - 12x + 36$

$0 = x^2 - 14x + 33$

$0 = (x - 11)(x - 3)$

$x = 11$ or $x \neq 3$

3. $\sqrt[3]{x} - 6 = -2$

$\sqrt[3]{x} = 4$

$x = 64$

E. Equations with Rational Exponents

1. $9(x - 7)^{\frac{4}{3}} = 9$

$(x - 7)^{4/3} = 1$

$x - 7 = 1$

$x = 8$

2. $(x + 1)^{\frac{1}{3}} = -2$

$(x + 1) = (-2)^3$

$x + 1 = -8$

$x = -9$

3. $64 = (m - 10)^{\frac{3}{2}}$

$64^{2/3} = m - 10$

$16 = m - 10$

$m = 26$

F. Quadratic Equations

1. $6x^2 + 7x - 3 = 0$

-18, $9 \cdot -2$, 7

$(2x + 3)(3x - 1) = 0$

$x = -\frac{3}{2}$ or $x = \frac{1}{3}$

2. $3x^2 - 4x - 3 = 0$

$x = \frac{4 \pm \sqrt{16 - 4(3)(-3)}}{6}$

$x = \frac{4}{6} \pm \frac{\sqrt{52}}{6}$

$x = \frac{2}{3} \pm \frac{2\sqrt{13}}{6}$

$x = \frac{2}{3} \pm \frac{\sqrt{13}}{3}$ or $x = \frac{2 \pm \sqrt{13}}{3}$

3. $4x^2 - 4x - 3 = 0$

-12, $-6 \cdot 2$, 4

$(2x + 1)(2x - 3) = 0$

$x = -1/2$ or $x = 3/2$

same.

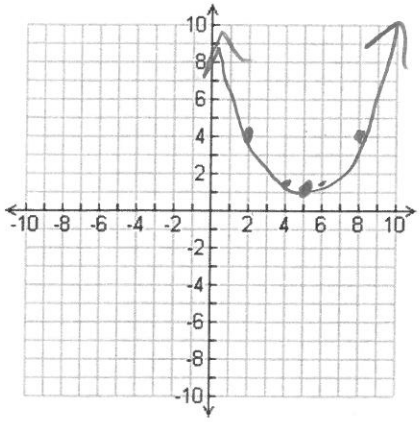
G. Graph

1. $y = \frac{1}{3}(x-5)^2 + 1$

Locator Point: (5, 1)

Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

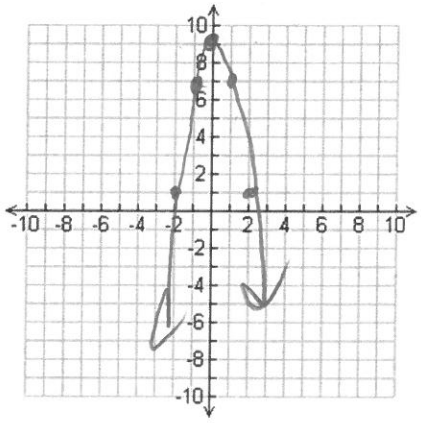


2. Graph $y = -2x^2 + 9$

Locator Point: (0, 9)

Domain: $(-\infty, \infty)$

Range: $[-\infty, 9]$



3. Graph $y = 4(3)^x - 2$ (0, 1) (1, base)

Locator Points: (0, 2) (1, 10)

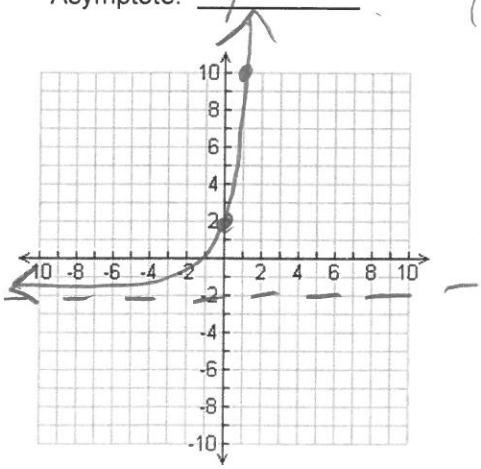
Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

Asymptote: $y = -2$

$\frac{2}{4} = 3^x$

$x =$



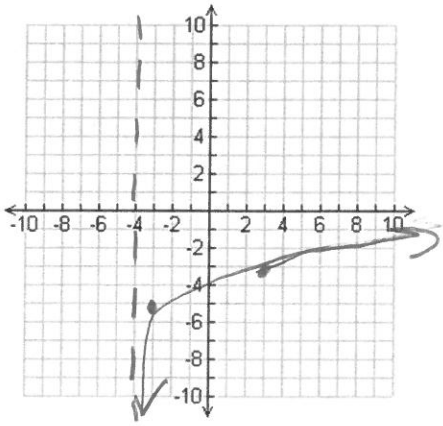
4. Graph $y = 2\log_7(x+4) - 5$ (1, 0) (base, 1) (-3, -5) (3, -3)

Locator Points: (-3, -5) (3, -3)

Domain: $(-4, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -4$

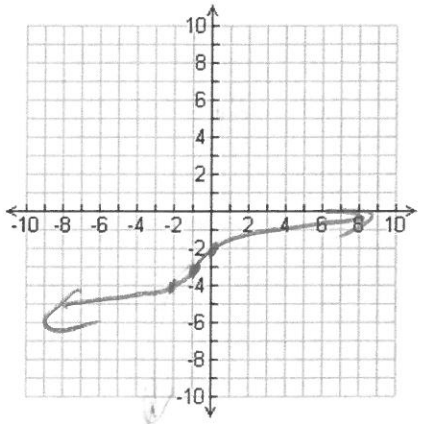


5. Graph $y = \sqrt[3]{x+1} - 3$

Locator Point: $(-1, -3)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



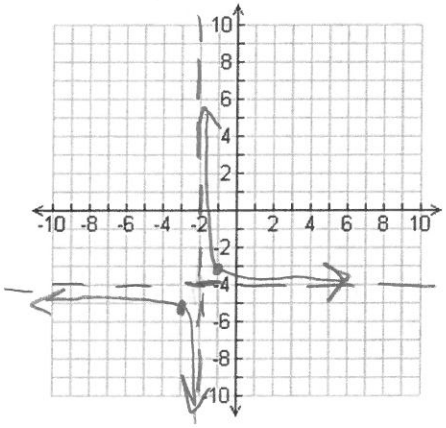
6. Graph $y = \frac{1}{x+2} - 4$

Locator Points: $(-1, -3)(-3, -5)$

Domain: $(-\infty, -2) \cup (-2, \infty)$ or $x \neq -2$

Range: $(-\infty, -4) \cup (-4, \infty)$ $y \neq -4$

Asymptotes: $x = -2$ & $y = -4$

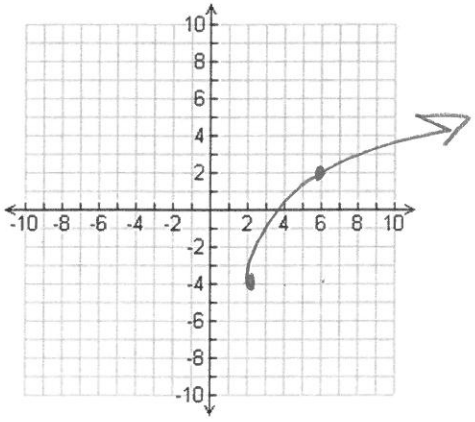


7. Graph $y = 3\sqrt[2]{x-2} - 4$

Locator Point: $(2, -4)$

Domain: $[2, \infty)$

Range: $[-4, \infty)$



$(-1, -4) \rightarrow (1, -6)$

$(1, 1) \rightarrow (5, -4)$

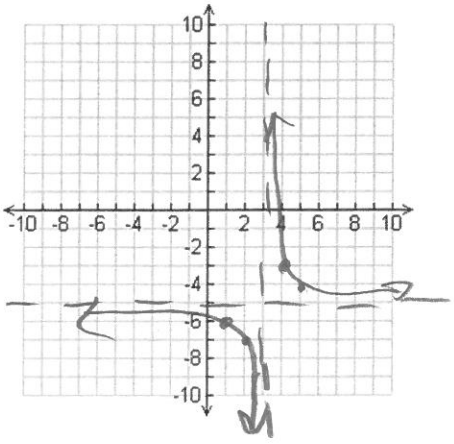
8. Graph $y = \frac{2}{x-3} - 5$

Locator Points: $(1, -6)(5, -4)$

Domain: $(-\infty, 3) \cup (3, \infty)$ or $x \neq 3$

Range: $(-\infty, -5) \cup (-5, \infty)$ or $y \neq -5$

Asymptotes: $x = 3$ $y = -5$

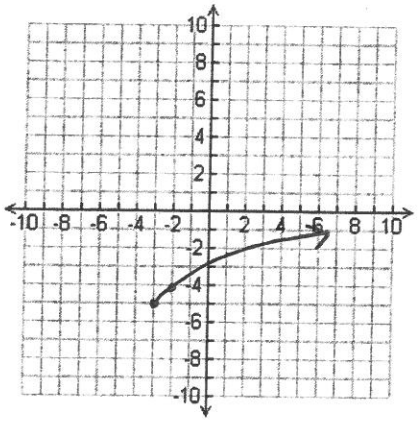


9. Graph $y = \sqrt{x+3} - 5$

Locator Point: $(-3, -5)$

Domain: $[-3, \infty)$

Range: $[-5, \infty)$



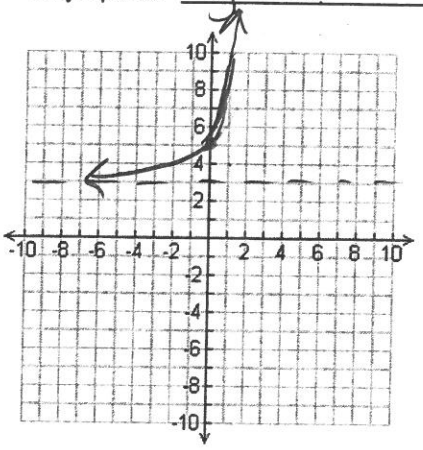
10. Graph $y = 2(5^x) + 3$

Locator Points: $(0, 5)$ $(1, 13)$

Domain: $\mathbb{R}$

Range: $(3, \infty)$

Asymptote: $y = 3$



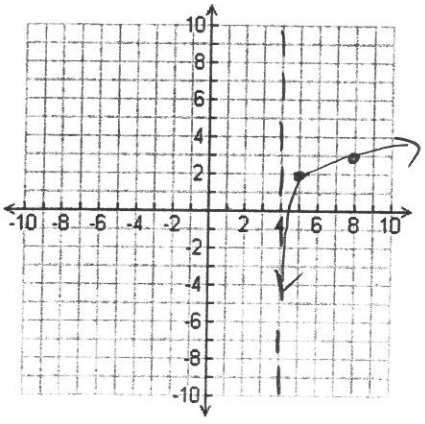
11. Graph $y = log_4(x-4) + 2$

Locator Points: $(5, 2)$ $(8, 3)$

Domain: $(4, \infty)$

Range: $\mathbb{R}$

Asymptote: $x = 4$

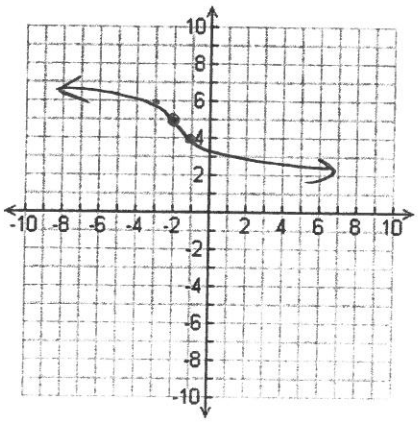


12. Graph $y = -\sqrt[3]{x+2} + 5$

Locator Point: $(-2, 5)$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$



art IV: Probability and Statistics Review for Final.

1. 8! 40320

2. ${}_{18}P_3$ 4896

3. ${}_{9}C_4$ 126

4. Simplify $\frac{6!}{3!}$ 120

5. Suppose you are asked to list, in order or preference, the three best movies you have seen this year. If you saw 10 movies during the year, in how many ways can the three best be chosen and ranked?

${}_{10}P_3 = 720$

6. An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?

${}_{6}C_3 = 20$

7. Seven bands have volunteered to perform at a benefit concert, but there is only enough time for four of the bands to play. How many lineups are possible?

${}_{7}P_4 = 840$

8. In the Long Beach Air Race six planes are entered and there are no ties, in how many ways can the first three finishers come in?

${}_{6}P_3 = 120$

9. A four-person committee is to be elected from an organization's membership of 11 people. How many different committees are possible?

${}_{11}C_4 = 330$

10. For the following sets of data, calculate the mean and standard deviation of the data. Describe the mean and standard deviation in words after calculating it.

a) The data set below gives the prices (in dollars) of cordless phones at an electronics store.

35, 50, 60, 60, 75, 65, 80

$\mu = 60.7$ $\sigma = 13.997$

b) The data set below gives the numbers of home runs for the 10 batters who hit the most home runs during the 2016 Major League Baseball regular season.

51, 48, 47, 46, 45, 43, 41, 40, 40, 39

$\mu = 44$ $\sigma = 3.82$

11. There are 3 red, 3 white, and 4 green marbles in a bag.

a) What is the probability that you will draw a green marble?

$P(G) = \frac{4}{10} = \frac{2}{5}$

b) You have successfully pulled out a green marble without putting it back in the bag. What is the probability that you will draw a red marble?

$P(R) = \frac{3}{9} = \frac{1}{3}$

c) All marbles are back in the bag. What is the probability that you draw 2 red marbles in a row if you replace the first one?

$P(R+R) = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100}$

d) All of the marbles are back in the bag. What is the probability that you choose 2 red marbles in a row without replacement?

$P(R+R) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

e) Again – all marbles are back again. What is the probability of choosing a green marble and then a white marble? (without replacement)

$P(G+W) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$

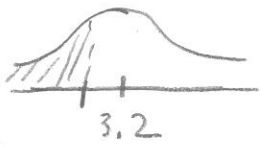
12. The diameters of oranges from Florida orchard are normally distributed with mean of 3.2 inches and standard deviation of 1.1 inches. A packing supplier is designing special occasion presentation boxes of oranges and needs to know the average diameter for a random sample of 8 oranges. What is the probability that the mean diameter for these oranges is

$\bar{X} = 3.2''$ $\sigma_{\bar{x}} = 1.1''$

a) Smaller than 3 inches?

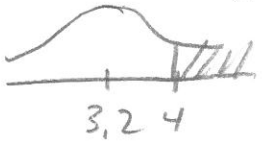
$P(x < 3'') = \text{Normcdf}(-\infty, 3, 3.2, 1.1)$

$\approx .428$



b) Longer than 4 inches?

$P(x > 4'') \approx .234$



c) Between 3 and 4 inches?

$P(3 < x < 4) \approx .339$

13. Suppose that a team of biologists has been studying the Pinedale children's fishing pond. Let x represent the length of a single trout taken at random from the pond. This group of biologists has determined that x has a normal distribution with a mean $\mu = 10.2$ inches and standard deviation $\sigma = 1.4$ inches.

a) What is the probability that a single trout taken at random from the pond is between 8 and 12 inches long?

$P(8 < X < 12) \approx .843$ Normcdf(8, 12, 10.2, 1.4)

b) What is the probability that a single trout taken at random from the pond is greater than 11 inches long?

$P(X > 11) \approx .284$ Normcdf(11, ∞, 10.2, 1.4)

14. In basketball, Nicole makes 4 baskets for every 10 shots. If she takes 3 shots, what is the probability that *exactly* 2 of them will be baskets?

$P = \frac{4}{10} = .4$ $n = 3$ $r = 2$

$P(2) = {}_3C_2 (.4)^2 (.6)^1 \approx .288$

15. Gordon tosses a fair die six times. What is the probability that he will toss *exactly* two 5's?

$P(2) = {}_6C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 \approx .201$ $n = 6$ $P = 1/6$ $r = 2$

16. Approximately 3% of the eggs in a store are cracked. If you buy two dozen eggs, what is the probability that

a) one of your eggs are cracked $P(1) = {}_{12}C_1 (.03)^1 (.97)^{11} \approx .258$

$n = 12$, $P = .03$ Mistake on answer key - see corrected answers below!

b) at least one of your eggs is cracked $P(r \geq 1) = 1 - P(0) = 1 - {}_{12}C_0 (.03)^0 (.97)^{12}$

$\approx .306$

c) exactly two of your eggs are cracked

$P(2) = {}_{12}C_2 (.03)^2 (.97)^{10} \approx .044$

Woops - I used 12 but it was 2 dozen so replace all 12s with 24 and the new answers are
a) .357 b) .519 c) .127